Invited Lecture

Language in Mathematics Education: Issues and Challenges

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ABSTRACT In this chapter I present some theoretical and methodological approaches on language in mathematics education. Language can be mainly viewed as the means to represent mathematical meanings or as constructing mathematical meanings itself. These approaches in turn lead to different methodologies, which in turn lead to different types of results. I argue that researchers should be cautious before adopting a particular theoretical framework, since sometimes frameworks are based on neighbouring concepts. I provide examples of such concepts, namely positioning and norm. The complexity of language and interaction in mathematics classrooms calls for more holistic and less dichotomised approaches. The combination of approaches is also an effective way to conduct research on language in mathematics education; examples of such combinations are provided.

Keywords: Language; Discourse; Norm, Positioning.

1. Introduction — Views on Language in Mathematics Education

The relationship between mathematics and language has been a topic of research for decades. It is worth noting that research in (mathematics) education and in language seem to share some common characteristics: both have initially focused on statistically and laboratory-based studies, but gradually have shifted to more interpretative and less taxonomical approaches, as Austin and Howson (1979) note. These authors were among the first to also note that the field of language in mathematics education deserves our attention, since "In the teaching and learning of mathematics, language plays a vitally important role" (p. 162).

So, a first question that we may ask is what is the role of language in mathematics education? Assuming that we view mathematics as precise and unambiguous (Ambrose, 2017), the role of language is communicating mathematical meanings among human agents. In that case, the effectiveness of communication resides on the agents' ability to code and decode the meanings entailed in language. However, our experience tells us that communication in the mathematics classroom is far from unproblematic and unambiguous. Rowland offers a characteristic example of a student's answer: "The maximum will probably be, er, the least 'll probably be' bout fifteen" (Rowland, 2000, p. 1). The mathematical proposition extracted from this

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sentence would be the following: "There are fifteen segments" (Tatsis and Rowland, 2006, p. 257). So, why would the student add hedges in his proposition or, in other words, why would he choose to make his contribution vaguer than expected? A short answer to this is because language in the mathematics classroom may be used to communicate more than mathematical concepts; in this case it is used to convey the speaker's uncertainty. Such situations, which are common in mathematics classrooms, are the object of studies which acknowledge that language is more than a 'neutral' communication system; it is a constitutive part of the world it describes and, concerning mathematics, there cannot be mathematics without language (cf. Derrida's claim that there is "nothing outside the text" (1976, p. 158)).

The idea of language in mathematics education seen as a social and interactive phenomenon has followed the shift from quantitative linguistics to sociolinguistics, pragmatics, discourse analysis and ethnomethodology (Schiffrin, 1994; see also Ingram (2018) for an overview on mathematics education). In mathematics education, we have witnessed a similar shift towards analyses of interactions (e.g., Krummheuer, 2007), where the focus can be on the establishment of a discursive community (Kieran et al., 2002), on the norms that regulate the verbal exchanges (Cobb and Yackel, 1996) or on the students' roles during the interactions (Tatsis and Koleza, 2006).

A common characteristic of these approaches is that they have adopted and eventually adapted concepts derived from sociology and social psychology. Two characteristic examples are the concept of norm and the concept of positioning, which will be discussed in the next section.

2. How Novel is your Approach? Neighbouring Concepts in Research

The big development of research in mathematics education — as witnessed by the increase of research papers, scientific journals and conferences — has led to a thrive of theoretical approaches. This has been also made possible by the easy access that researchers have to a multitude of studies worldwide, coming from different disciplines. Interdisciplinary and transdisciplinary approaches have been also made possible by the modern means of communication and exchange of information. Researchers, especially the younger ones, usually find themselves in the position of having to choose between an existing framework to ground their work or create a new one; their most frequent choice is presented in the next quote:

It has become the norm rather than the exception for researchers to propose their own conceptual framework rather than adopting or refining an existing one in an explicit and disciplined way. This prolific theorizing might be represented as the sign of a young and healthy scientific discipline. But it may also mean that theories are not being sufficiently examined, tested, refined and expanded. (Editors of Educational Studies in Mathematics, 2002, p. 253)

So, we, as researchers or reviewers, find ourselves in the position to examine the actual novelty of an approach, but mostly how well it is scientifically grounded. In order to do so, we need to sometimes study the historical background of the proposed theoretical constructs. I present two examples related to my own research, in order to highlight the similarities among concepts that are met in research with different names.

2.1. Norm and neighbouring constructs

The concept of norm, especially the subconstructs of the social and the sociomathematical norm, is a characteristic example of a concept which was successfully transferred from sociology to mathematics education. This was mostly done by the work of Paul Cobb, Erna Yackel and their colleagues (e.g., Cobb and Yackel, 1996; Yackel and Cobb, 1996; Yackel et al., 2000). The social norms are related to the structure of classroom activity in general and may include

> … explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations in which a conflict in interpretations or solutions had become apparent. (Cobb and Yackel, 1996, p. 178).

The sociomathematical norms are related to mathematical activity and refer to which contribution counts as "a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (Cobb and Yackel, 1996, p. 179). The origins of the concept of norm can be traced in the concept of prescription, as described, e.g., in the work of Biddle and Thomas (1966), who define prescriptions as

> behaviours that indicate that other behaviours should (or ought to) be engaged in. Prescriptions may be specified further as demands or norms, depending upon whether they are overt or covert, respectively. (p. 103)

In mathematics education research, one may find few more concepts, which seem related to the concept of norm. The first one is the concept of obligation, which, according to Voigt (1994), is an interactive construct that connects various routines in the mathematics classroom: teacher's and students' actions are constrained by some obligations, and this may especially become apparent in cases of conflict. The second concept is the meta-discursive rule, introduced by Anna Sfard and refers to "mostly tacit navigational principles that seem to underlie any discursive decision of the interlocutors" (2002, p. 324). Sfard (2008) goes on to claim that a rule is considered a norm only if it fulfils two conditions: it must be widely enacted within the discursive community and it must be endorsed by almost all members of that community, especially those considered as experts. The third concept which seems to be related to the concept of norm is the didactical contract, as introduced by Brousseau (1997): it refers to specific habits of the students that are expected by the teacher and vice-versa. Knowledge construction is seen as a shared responsibility between the teacher and the students, and as a result of interpretations of each other's actions. However, an established didactical contract can also create problems for the students, especially when they enter a situation where the contract changes considerably, e.g., during the transition from primary to secondary education or from secondary to tertiary education.

2.2. Positioning and neighbouring constructs

Positioning theory, as Herbel-Eisenmann et al. (2015) claim, is another example of an imported theory in mathematics education. Originated in the work of Rom Harré and his colleagues (e.g., Harré and van Langenhove, 1999), the theory has been deployed in mathematics education in order to interpret communicative actions performed by the teachers and the students. Such actions affect the positioning of each other in the establishment of mathematical knowledge, by sometimes challenging the prevailing authority structures. For example, it is assumed that the teacher is an authority of the classroom; what happens if this authority is challenged by the teacher herself? In a next section I will demonstrate how this approach was combined with another sociological approach, in order to analyse interactions. At this point though, I will present another concept, which is neighbouring to positioning: the concept of framing, introduced by Goffman (1974). This concept is part of a wider system of constructs used to interpret how people interactively define the situations they are involved in. Particularly, during any interaction, speakers align between each other, but also in relation to utterances; this creates the space for intersubjective knowledge.

The next neighbouring concept is the interactive frame, proposed by Tannen and Wallat (1993) and refers to "a definition of what is going on in interaction, without which no utterance (or movement or gesture) could be interpreted" (pp. 59–60). By moving to a linguistic approach, we find Gumperz's (1982) notion of speech activity. According to Gumperz, the analysis of framing can be done by the use of contextualisation cues, which are defined as "any feature of linguistic form that contributes to the signalling of contextual presuppositions" (1982, p. 131).

Summing up, the relationships — or the borders — between the aforementioned constructs are not clearly defined; Gordon (2015) presents examples of researchers who consider them as rough synonyms (Tannen, 1994); she also claims that the notion of storyline can bring together positioning and framing:

> framing — growing from a field that has tended to look out into the world (sociology) — and positioning — developing from one that has tended to look within (psychology) — have found a meeting point, where the interactional and the psychological are understood as inseparable in language. (p. 340)

Until now I have claimed that neighbouring concepts, originating in other disciplines, are deployed in various studies, including studies in mathematics education. In the next section, I present another characteristic of studies on language in mathematics education: the use of (supposedly) clearly defined categories to describe and distinguish the relevant phenomena.

3. Distinctions and Dichotomies that We Live By

One may claim that it is apparent for researchers to deploy clearly defined categories in order to better present their results to the scientific community and the general public.

If we limit ourselves to the relationship of language and mathematics, we may notice a basic distinction between mathematical and everyday language. The former has specific vocabulary and syntax, minimum use of verbs and an absence of the human agent:

> … the prevailing image of mathematical writing, perpetuated in most of the texts encountered by students in the later years of schooling and at university, is still impersonal, lacking a narrative of human involvement in doing mathematics. (Morgan, 2001, p. 169) [2]

The above distinction, which refers to written texts, might have served research well during the initial attempts to analyse the linguistic phenomena in mathematics education. However, all contemporary researchers agree that language needs to be viewed as a multi-faceted phenomenon:

> Analyses should consider every-day and scientific discourses as interdependent, dialectical, and related rather than assume they are mutually exclusive. (Moschkovich, 2018, p. 40)

Following the above, there have been attempts to overcome the dichotomy of ordinary versus mathematical language. A characteristic example is the categorisation suggested by Pirie (1998) for the means of mathematical communication: ordinary language, mathematics verbal language, symbolic language, visual representations, unspoken shared assumptions, quasi-mathematical language. Pirie (1998) goes on to discuss the problems that occur during the move from ordinary to mathematical language, by presenting relevant examples from solving linear equations to division. She also claims that problems also exist during the move from verbal mathematical to symbolic language. Then she discusses two of the most interesting parts of her framework: the unspoken assumptions shared by the students (which resembles the concept of norm discussed before) and the quasi-mathematical language, which is:

> … carrying meaning for the users that is unorthodox or incompatible with acceptable mathematical language. On the other hand, however, this quasi-mathematical language leads to no difficulties in understanding and can, in fact, often enhance understanding by forming a language-linked image that is of personal relevance to the learner. (p. 22)

The existence of quasi-mathematical language, and its role in establishing shared mathematical meanings has been examined in a series of studies that I have done in the last years. The core of these studies has been a game called 'Broken phone', in which the students are asked to either describe (based on a drawn image) or draw (based on written instructions) complex geometrical figures (see Fig. 1).

[2] The above stereotypical view is challenged by most — if not all — researchers today. For instance, Morgan, in the paper quoted, suggests widening the spectrum of what is considered 'appropriate' and 'mathematical' by the teacher, leaving room for students' descriptions of their investigations and their problem solving processes.

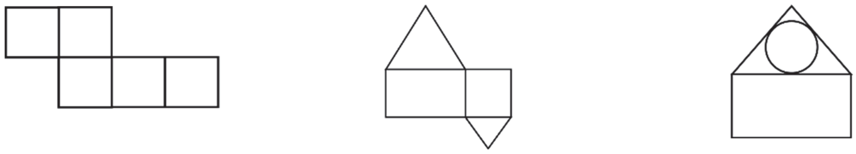

Fig. 1. Figures given in the 'broken phone' game (Tatsis, 2007; Tatsis and Moutsios-Rentzos, 2013)

One of the results of these studies has been that the students have sometimes been able to communicate the properties of the given figures by using either everyday or quasi-mathematical language.

A second way to overcome the unproductive distinctions while studying language in mathematics education, is by combining theoretical and even methodological frameworks. This possibility is discussed in the next section.

4. The Need to Combine Approaches

It has been apparent from the previous sections that language in mathematics education is a rich, complex and evolving field of mathematics education. Moreover, due to its focus on language and interactions, it has been importing or adapting theories and methodologies from linguistics, sociology and psychology. Some researchers in the field have realised that each theoretical approach is good enough to shed light on only one (or few) sides of the multi-faceted phenomena of mathematics teaching and learning. That is why the need for combining theories in a systematic way (usually referred to as networking theories, see, e.g., Bikner-Ahsbahs et al., 2014) has come to the fore. The advocates of this approach acknowledge that the variety of theories is a resource that researchers should build upon, but at the same time, a unified 'theory of everything' is far from possible (Prediger et al., 2008).

Despite the above suggestions, the instances of combined approaches are still scarce. As I mentioned before, many researchers are focused on introducing a 'novel approach' that 'will shed new light' on the phenomena of interest, sometimes neglecting the significance of existing theories, which become even more powerful when combined. A characteristic example comes from combining positioning theory, that was mentioned before, with politeness theory (Brown and Levinson, 1987) for the analysis of a mathematics teacher's interactions with his students. Tatsis and Wagner (2018) have presented two juxtaposed analyses of particular excerpts, which were originally analysed elsewhere (Wagner and Herbel-Eisenmann, 2014) by the lens of positioning, particularly in relation to the authority structures that have been observed: personal authority, discourse as authority, discursive inevitability, and personal latitude. Tatsis and Wagner then enriched these juxtaposed analyses with a combined one. According to this, a student's question (which, according to positioning theory, was classified as a manifestation of the student's personal latitude) was also seen as threat to the teacher's positive face. This, in turn, allowed the authors to interpret the teacher's reaction as an attempt to protect his face. As a result, the combination of these approaches has allowed the authors to gain the insights from both analytical lenses:

> … the two theories are both interested in the phenomena that occur during classroom interactions; moreover, we see them as complementary since politeness theory helps us consider reasons for teachers and students to choose particular authority structures in their classroom interactions. Thus, we generally believe that in order to fully comprehend the dynamics of the exchanges in the mathematics classroom we need to be able to continuously shift our focus from the participants' acts to the established (or striving-to-be-established) norms and from the participants' positionings to their own and the others' face-wants. (pp. 183–184)

At this point, it is worth mentioning that there have also been attempts to offer juxtaposed (but not combined) analyses of the same data; such attempts may also have a significant contribution to research on language in mathematics education. A characteristic example is the paper of Candia Morgan and her colleagues (Morgan et al., 2007), in which six researchers analysed a given excerpt by Cohors-Fresenborg and Kaune (2007), by deploying six different theoretical and methodological approaches.

5. Discussion and Recommendations for Research

In the present chapter, I have drawn upon various approaches for studying phenomena related to language in mathematics education. I did not aim to cover all approaches, since such works already exist in the literature (e.g., Planas et al., 2018); my aim was to focus the readers' attention on particular issues that I consider significant in the field. The first issue is the neighbouring theoretical concepts, which usually originate from sociology, psychology or linguistics, but all refer to the same phenomenon. I have provided the examples of two such concepts, which are deployed in studies of language and interactions in mathematics education, namely the norm and positioning. I claim that one way to deal with this is to deeply examine the background of such concepts — see their differences and their similarities, be cautious before adopting one and restrain oneself from giving a new name to an already existing construct. Neologisms seem attractive, but most of the times lead to confusions and repetitive studies.

The second issue refers to the distinctions, and eventually the dichotomies that appear in some studies, the most prominent being the one between everyday and mathematical language. Although these can be helpful for particular kind studies, I claim that it is more realistic to view language as a more complex and more dynamic phenomenon. In contemporary multilingual and multimodal mathematics classrooms, such dichotomies fail to grasp the complexity of the verbal and non-verbal interactions. Therefore, we need more holistic approaches — which may come from the combination of existing approaches; such approaches may provide the researchers with the appropriate tools to zoom in and out from the micro-level of face-to-face interactions to the meso-level of the classroom discursive community (and the interpersonal relations) and then to the macro-level of the school and education system and their discursive practices. A second way to deal with such dichotomies is by establishing new categories, eventually placed between the existing ones, in order to cover the 'grey zones' that exist in real classroom interactions. I have provided an example of a study, in which quasi-mathematical language was effectively used to communicate geometrical concepts.

The last issue refers to the need to combine approaches in the study of language in mathematics. As I mentioned above, this need is grounded on the need to analyse in a holistic way the phenomena related to language in mathematics education. I have provided an example of such a study, in which positioning theory and politeness theory were effectively combined to offer new insights to an already-analysed episode. There is a movement in mathematics education (networking theories) that focuses on combining particular theoretical approaches. I claim that it is generally a positive thing that more combination studies appear in the field; only by allowing ourselves to view phenomena from different points of view, we can achieve a wider understanding of language and mathematics teaching and learning.

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